JOURNAL OF MATERIALS SCIENCE 12 (1977) 1421–1425

Lifetime prediction of brittle materials having spatial variations in fracture properties K_{IC} and v versus K_I

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The lifetimes of brittle materials under conditions of subcritical crack growth are analysed to indicate the effects of spatial variations of K_{IC} and v versus K_I and methods for studying the variations are suggested.

1. Introduction

The theory of fracture mechanics permits the calculation of a minimum service life for a brittle material exhibiting subcritical crack growth under known loads and environmental conditions [1–3]. This requires proof testing to establish, a_u, the upper limit for crack size, knowledge of the critical stress intensity factor, K_C, and the dependence of the crack front velocity, v, on the stress intensity factor, K.

The above considerations involve the assumption that the elastic and fracture properties are spatially homogeneous and continuous, but it is well known that this assumption becomes invalid as the dimensions of a volume element under consideration are reduced [4–10]. It is also well known that v versus K_I diagrams based on long crack fronts do not always agree with the indications of measurements that depend on relatively short crack fronts [4–6]. For example, the parameter n determined in fitting data to the form $v \propto K_I{}^n$ may be substantially different when determined by a double torsion or a double cantilever beam technique [3] than when determined by analysing the loading rate dependence of strength [4, 5].

Some physical manifestations of inhomogeneous fracture mechanics properties are discontinuous crack growth under continuous loading [6, 7], acoustic emission peaks [8], crack branching, and other forms of deviation from ideal crack propagation in cases where the stress and initial crack geometry are simple.

The purpose of this paper is two-fold: first, to demonstrate by a simplified analysis the possibility that large variations in predicted lifetimes may result from spatial variations in fracture mechanics properties; and second, to suggest the possibility of improved methods of determining the constancy or variability of the fracture mechanics properties of materials.

2. Analysis

Two basic equations relevant to crack growth in ceramics and glass are

$$K_I = \sigma Y \sqrt{a} \tag{1}$$

where Y is a dimensionless factor which depends on the geometry of the crack and specimen, K_I is the stress intensity factor for mode I opening, σ is the applied stress, and a is a flaw crack length;

$$v = AK_I{}^n \tag{2}$$

where v is the velocity with which a crack front moves, and A and n are experimentally determined constants for a range of K_I values under conditions of fixed temperature and environment. The time to failure from a flaw with an initial length, a_i, under an applied stress, σ, as derived from Equations 1 and 2 is then [1, 3]:

$$t = 2K_{\mathrm{Ii}}^{2-n}/[A\sigma^2 Y^2(n-2)] \quad (3)$$

where K_{Ii} is the initial stress intensity factor at the flaw.

By proof testing with a stress σ_{p}, one can select specimens conforming to the condition

$$K_{\mathrm{IC}} > \sigma_{\mathrm{p}} Y\sqrt{a_{\mathrm{i}}} \quad (4)$$

where a_{i} is a length of any flaw surviving the proof test condition, and K_{IC} is the critical stress intensity factor. The minimum possible time to failure under an applied stress, σ, is then [1, 3]:

$$t_{\mathrm{m}} = 2(K_{\mathrm{IC}}\sigma/\sigma_{\mathrm{p}})^{2-n}/[A\sigma^2 Y^2(n-2)]. \quad (5)$$

We now consider a case where a flaw is imnedded in a localized atypical region having elastic and fracture properties different from the bulk properties. In this case the stress intensity factor for the localized region may be written as

$$K'_{\mathrm{I}} = F(a', x'_{\mathrm{i}}, \beta'_{\mathrm{j}}, \beta_{\mathrm{k}}, \sigma) \quad (6)$$

where F is a function of the crack length, other geometric variables, and the elastic constants a', x'_{i}, and β'_{j}, respectively, for the atypical region, of the elastic constants of the bulk, β_{k}, and of the applied stress. One can also assume that the sub-critical crack growth in the atypical region is also described by the form

$$v' = A'K_{\mathrm{I}}'^{n'}. \quad (7)$$

In order to arrive at the functional form of Equation 1, one can write

$$F(a', x'_{\mathrm{i}}, \beta'_{\mathrm{j}}, \beta_{\mathrm{k}}, \sigma) \equiv Y\sigma\sqrt{a'}/f(a', x'_{\mathrm{i}}, \beta'_{\mathrm{j}}, \beta_{\mathrm{k}}, \sigma) \quad (8)$$

so that

$$K'_{\mathrm{I,eff}} \equiv K'_{\mathrm{I}}f(a', x'_{\mathrm{i}}, \beta'_{\mathrm{j}}, \beta_{\mathrm{k}}, \sigma) = Y\sigma\sqrt{a'}. \quad (9)$$

It is possible to work with the effective stress intensity factor defined in Equation 9, but instead we will assume that $f(a', x'_{\mathrm{i}}, \beta'_{\mathrm{j}}, \beta_{\mathrm{k}}, \sigma) = 1$, which corresponds to a condition of continuous invarient elastic properties. We also assume that the properties change in a stepwise manner at the boundary.* Thus, we obtain

$$K'_{\mathrm{I}} = Y\sigma\sqrt{a'} \quad (10)$$

and

$$K'_{\mathrm{IC}} > \sigma_{\mathrm{p}} Y\sqrt{a'_{\mathrm{i}}} \quad (11)$$

which corresponds to Equations 1 and 4. If we further define a_{u} and a'_{u} as upper bounds for a_{i} and a'_{i} as determined by a proof test and K_{Iu} and K'_{Iu} as upper bounds for K_{Ii} and K'_{Ii}, respectively, under an applied load, σ, then it follows that

$$(a'_{\mathrm{u}}/a_{\mathrm{u}})^{1/2} = K'_{\mathrm{Iu}}/K_{\mathrm{Iu}} = K'_{\mathrm{IC}}/K_{\mathrm{IC}}. \quad (12)$$

Equations 7 and 10 permit us to calculate the time to failure of a flaw as illustrated in Fig. 1, where for simplicity the flaw is placed at the centre of a spherical atypical region with a radius R:

$$t' = (2/\sigma^2 Y^2)\int_{K'_{\mathrm{Ii}}}^{K''_{\mathrm{I}}}(K'_{\mathrm{I}}/v')\mathrm{d}K'_{\mathrm{I}} + \int_{K''_{\mathrm{I}}}^{K_{\mathrm{IC}}}(K_{\mathrm{I}}/v)\mathrm{d}K_{\mathrm{I}} \quad (13)$$

$$t' = (2/\sigma^2 Y^2)\,[(K_{\mathrm{Ii}}'^{2-n'} - K_{\mathrm{I}}''^{2-n'})/A'(n'-2) + (K_{\mathrm{I}}''^{2-n} - K_{\mathrm{IC}}^{2-n})/A(n-2)], \quad (14)$$

where

$$K''_{\mathrm{I}} \equiv \sigma_{\mathrm{a}} Y\sqrt{R}. \quad (15)$$

The ratio of minimum times to failure for flaws in the atypical and typical regions is

$$t'_{\mathrm{m}}/t_{\mathrm{m}} = \frac{(K_{\mathrm{Iu}}'^{2-n'} - K_{\mathrm{I}}''^{2-n'})/A'(n'-2) + K_{\mathrm{I}}''^{2-n}/A(n-2)}{K_{\mathrm{Iu}}^{2-n}/A(n-2)}; \quad (16)$$

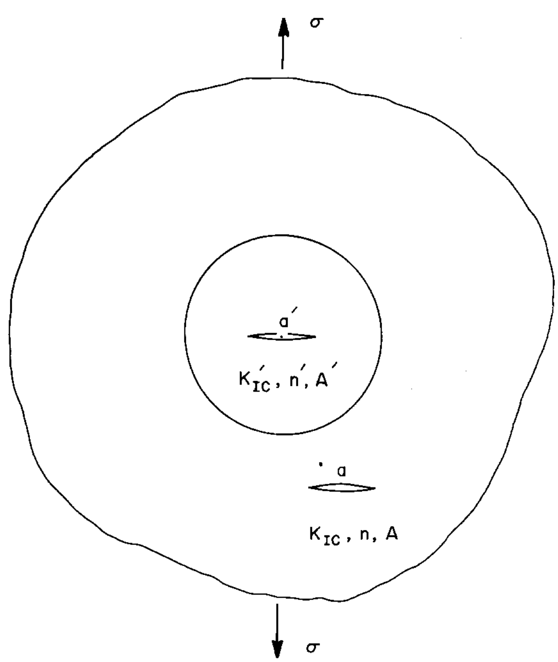

Figure 1 Illustration of cracks in atypical and typical regions.

*The assumption of stepwise behaviour is made here as a mathematical convenience only. If does not reflect expected material behaviour in general, although it may apply to cases such as a flaw within a large single grain.

the terms involving K_{IC} were dropped from Equation 16, as from Equations 3 and 5, because of relative insignificance. Furthermore, if R is greater than $2a_i$ or $2a_p$ and $n' > 10$, the term $K_I''^{2-n'}$ is negligibly small in Equations 14 and 16. Equations 14 and 16 have two time intervals, one where $a' < R$ and one where $a' > R$, and either term may dominate. If R happened to be larger than the critical crack size, Equations 14 and 16 would be replaced by dropping all terms except the one containing K'_{Ii}.

If the only atypical property is K'_{IC}, Equations 12 and 16 provide that

$$t'_m/t_m = (K'_{IC}/K_{IC})^{2-n}. \tag{17}$$

In this case, Fig. 2 shows that minimum lifetimes decrease rapidly with increasing K'_{IC}; e.g. $t'_m/t_m < 0.01$ for a K'_{IC}/K_{IC} ratio of 1.15 if $n = 40$ and a ratio of 1.8 if $n = 10$. The decreased lifetimes result from increases in the flaw sizes that can survive a proof test, and then extend under subsequent conditions for subcritical crack growth.

Another simplification results if it is assumed that $K'_{IC} = K_{IC}, R > 2a'$, and $n' > 10$. That is,

$$t'_m/t_m = A(n-2)/A'(n'-2) + (K'_{Iu}/K''_I)^{n-2} \tag{18}$$

or

$$t'_m/t_m = A(n-2)/A'(n'-2) + (a'_u/R)^{(n-2)/2} \tag{19}$$

In most cases the last term should be $\ll 1$ and, therefore, negligible unless the first term is correspondingly small.

3. Discussion

It is important to recognize that most experimental studies of fracture and elastic properties begin with the objective of determining and evaluating the bulk properties of materials as though they are constant and continuous for fixed conditions. Therefore, in the calculation of lifetimes for $v-K$ diagrams, the material properties are treated as constants [1–3]. However, if Equation 14 of our analysis is written in terms of a_u and R, one can easily see that most of the time between initial loading and failure due to subcritical crack growth is consumed while the crack remains less than twice the original size. Therefore, one is concerned with small volumes, (20 to 200 μm)^3 typically, in the analysis of fracture resulting from slow crack growth. Volumes of this size are known to behave differently [4–10, 12]. In the analysis of reliability of lifetime predictions, the errors in measurements of the fracture properties have been considered [11, 12], but so far the effects of real spatial variations have not. The importance of the latter are stressed here by consideration of $K'_{I,eff}$, the effective stress intensity for a particular defect.

We have determined that lifetimes under conditions of subcritical crack growth may be quite sensitive to spatial variations in K_{IC}. Larger than average values permit correspondingly large defects

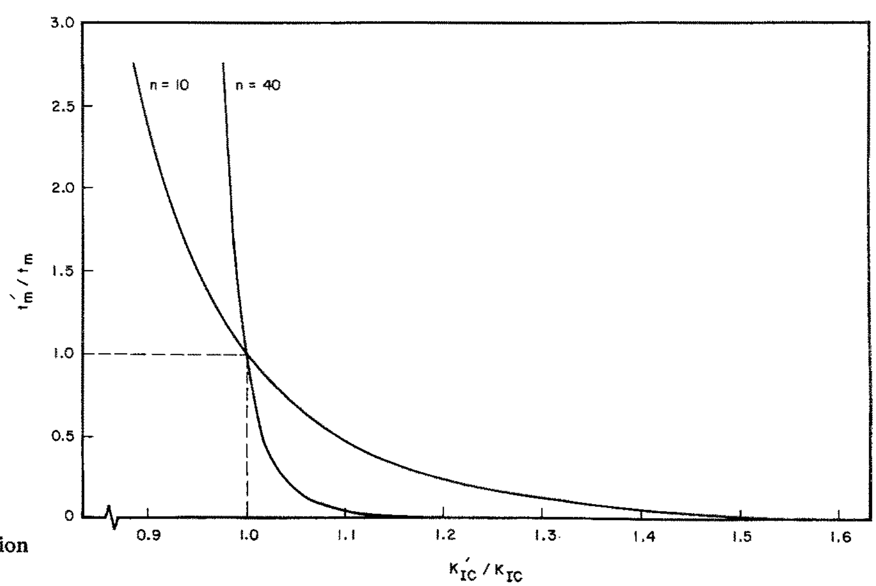

Figure 2 A plot of Equation 17 for $n = 10$ and $n = 40$.

to survive proof tests and the large defects may then severely limit lifetimes. It is reasonable to expect somewhat similar effects for some combination of geometry and local atypical elastic properties. One example is a change in local stress intensity with a change in temperature following proof testing at a lower temperature. The effect of variations in A and n under simplifying assumptions is illustrated in Equations 18 and 19. Some spread in the v–K curves is expected [6], but at this time the magnitude of variations is unknown for most materials.

Variations in the measured fracture mechanics properties, K_{IC} and v versus K, may correspond to flaw types [9], regions of atypical grain size, abnormally large grains, composition gradients surrounding inclusions [10], poor mixing, segregation of minority phases, localized residual stress, or flaw interactions. If the variations are appreciable or unknown, it is unlikely that proof testing can provide reliable minimum lifetimes for use conditions [11]. Instead it may be necessary to rely on lifetime expectations where these are based on statistical variations of the fracture properties as well as flaw sizes.

More efficient and economical methods of determining the constancy or variability of fracture properties are needed; some are suggested by the work of Mendiratta and Petrovic [13] and by Petrovic *et al.* [14, 15] where K_{IC} values are determined through the use of Knoop indentations and associated semi-elliptical cracks. Mendiratta and Petrovic showed that $K_{IC} \propto P/z_0\sqrt{a}$, where P is the indenter load, z_0 is the plastic deformation depth, and a is the crack depth. This result or a possible refinement may present the possibility of determining variations in K_{IC} in material volumes near the surface of a single specimen from the variation in the size of indent flaws for fixed loads.* In addition, indentation-produced cracks have been observed to extend in an approximately semi-elliptical configuration under four-point bending and conditions of subcritical crack growth; if it is established that the semi-major axis, $2c$, measured at the surface is in agreement with that determined from fracture surfaces, then the variations in the v–K_I parameters, A and n, can be determined from incremental extension of the crack. This follows from the analysis in the Appendix. The advantage of this approach is that a single specimen having a large number of controlled initial cracks can be utilized to generate a set of data yielding the v–K_I dependence of each cracked region. A broad range of crack lengths and velocities can thus be studied efficiently. The technique is a possible alternative to v–K_I determinations using the loading rate method, which requires many test bars to determine a value of n appropriate to small volumes [4, 5]. Finally, it provides for direct statistical study of the time for a flaw to grow to critical size.

Appendix

Knoop indentation of SiC and Si_3N_4 causes a nearly semicircular crack to form beneath the indent. When the crack is extended in a four-point bend fixture, which produces a stress that decreases with depth, the crack becomes semi-elliptical with a regularly decreasing a/c ratio, where a and c are the semi-minor and semi-major axes, respectively. In this case one can show that the stress intensity factor,

$$K_I = \sigma M(\pi a/Q)^{1/2} \tag{A1}$$

where $M \approx 1.03$ if $a \ll h$, the specimen thickness [14], can be written as

$$K_I \approx 2\sigma M(c/\pi)^{1/2} \equiv \sigma Y\sqrt{c}. \tag{A2}$$

This follows because Q,

$$Q \cong \phi^2 = \left[\int_0^{\pi/2} (\sin^2\phi + a^2\cos^2\phi/c^2)d\phi\right]^2, \tag{A3}$$

has an approximately linear dependence on a/c in the range $0.7 < a/c < 1$ [16], so that

$$Q \cong \pi^2/4 - 2.2(1 - a/c) \cong \pi^2\, a/4c. \tag{A4}$$

This permits us to proceed with the following derivation:

$$dc/dt = v = AK_I^n = A(\sigma Y)^n c^{n/2}. \tag{A5}$$

Integrating and rearranging yields

$$c/c_0 = [1 - c_0^{(n-2)/2}\, A(\sigma Y)^n t(n-2)/2]^{2/(2-n)}, \tag{A6}$$

or

$$c/c_0 = (1 - Zt)^{2/(2-n)}, \tag{A7}$$

where

$$Z \equiv AK_{Ii}^n(n-2)/2c_0, \tag{A8}$$

*We recognize, of course, that more general information on the variation of K_{IC} must come from study of those accidental flaws which give rise to fracture at elevated temperatures in appropriate atmospheres.

and where c_0 is the initial value of c. Using Equations A2, A5, and A7 we obtain

$$v = AK_{\mathrm{Ii}}^{n}(1 - Zt)^{n/(2-n)} \qquad \text{(A9)}$$

$$K_1 = K_{\mathrm{Ii}}(1 - Zt)^{1/(2-n)}. \qquad \text{(A10)}$$

Equation A7 can be expressed as

$$\log(c/c_0) = 2 \log (1 - Zt)/(2 - n), \qquad \text{(A11)}$$

which permits one to use any set of three c and t values from a curve of c versus t to solve for A and n. Thus, if we write

$$f \equiv \log(c_1/c_0)/\log(c_2/c_0), \qquad \text{(A12)}$$

we may write

$$(1 - Zt_1) - (1 - Zt_2)^f = 0. \qquad \text{(A13)}$$

This is solved for Z and then Equations A11 and A8 are used to determine n and A. Equations A9 and A10 can then be used to compute v and K values if desired. Finally, if it is assumed that the crack growth behaviour is described for all values of c by one set of A and n values, the time to failure is given by $t_f = Z^{-1}$.

One can express the factor M in Equation A1 as

$$M = 1.03(1 - 1.25\, c/h)$$

where h is the thickness of a four-point bend specimen and where $a/c \approx 1$. If this function were incorporated into Equation A5, one could not derive an explicit equation for c, v, or K_{I}. Since errors in c and v are quite sensitive to errors in K_{I} that result with increasing c/h, it is desirable to keep the initial c/h values as small as possible while maintaining the indentation-produced cracks as the dominant defects. We note that the cracks for small indents may not extend to the ends of the indent, and that the shape of the initial crack might be slightly different than the crack after growth in four-point bending. For these reasons the initial crack lengths to be used in calculations should normally be for cracks which have been extended from the original lengths by small amounts. The lengths of cracks one can analyse are, of course, limited by the thickness and width of specimens.

In measuring crack lengths, the accuracy obtainable from an optical microscope with bright-field illumination may be increased by using an ultra-violet stimulated dye penetrant having visible fluorescence. We presume that the ultimate accuracy could be achieved by using a scanning electron microscope with a micrometer stage. We also note that oxidation products formed while a speciment is cracked in an oxidizing atmosphere may contribute to ease and accuracy of crack measurement. The edge of specimens should be inspected to ensure that finishing was sufficient to have prevented appreciable crack growth from machining flaws at the edges.

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Received 25 August and accepted 22 November 1976.